Estimation of the term structure of interest rates for Moroccan financial market using Vasicek model

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Abstract: *The term structure of interest rates measures the relationship among the yields on default-free securities that differ only in their term to maturity. This paper is an attempt to estimate the term structure of interest rates using the Vasicek model as a one-factor model which considered the short interest rate as a risk. Our data is a weighted average rate from the Moroccan money market considered as a short rate. The choice of the short interest rate is owned from the choice of the one-factor model of interest rates. To model the term structure of interest rates we use the ordinary least square (OLS) and then we estimate the Vasicek model's parameters. Finally, we construct the yield curve of Moroccan financial market.*

Key words: *term structure of interest rates, Vasicek model, one-factor model.*

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Introduction:

The term structure of interest rates measures the relationship among the yields on default-free securities that differ only in their term to maturity. The determinants of this relationship have long been a topic of concern of economists and practionners. The term structure embodies the market's anticipations of future events.

Modeling the term structure of interest rate represents one of the most important topics on financial research. Since the introduction of interest rate dependent assets, an attention has been given to the development of models to price and hedge interest rate dependent assets or to manage the risk of interest rates contingent portfolios.

There are many models, which have been used to model the term structure of interest rates. In this paper, one model is considered: the Vasicek model. All those models are based on the previous theory of term structure. However, in a certain context, equilibrium forward rates must coincide with future spot rates, but when uncertainty about future rates is introduced the analysis become much more complex, theories of the term structure have taken the certainty model as their starting point and have proceeded by examining stochastic generalizations of the certainty equilibrium relationship. Merton (1973), Brennan and Schwartz (1977), and Vasicek (1977) have all assumed that the instantaneous spot rate of interest rate follows a gauss-wiener process. Those models are called single-factor models, which incorporate one state variable. A review of literature of some of those models is shown. A partial listing of these interest rate models includes those by Merton (1973), Brennan and Schwartz (1977, 1979, 1980), Vasicek (1977), Dothan (1978), cox, ingersoll and ross (1980,1985), longstaff (1989), hull and white (1990), black and karsinski (1991), and longstaff and Schwartz (1992).

The main question in this paper is " does the Vasicek model of term structure of interest rates fit very well the yield curve of Moroccan financial market?".

An econometric analysis of the term structure interest rates is shown using data from Moroccan money market in order to model the Vasicek model for this market. Ordinary least square method is used in this paper to estimate the Vasicek model parameters.

The plan of this paper is as follows. Section 1 presents a review of literature of the major one- factor models of the term structure of interest rates. In Section2, we summarize the Vasicek model. Section 3, the parameters of the assumed stochastic process for interest rates are estimated using data on Moroccan interest rates.. Section 4, a comparison between empirical and real observed yield curve . section 5: we conclude.

Section 1: review of literature of one-factor models of term structure of interest rates:

Several models of the term structure have been proposed in the academic literature. Examples are Brennan and Schwartz (1979, 1982), Courtadon (1982), Cox, Ingersoll, and Ross (1985b), Dothan (1978), Langetieg (1980), Longstaff (1989), Richard (1979), and Vasicek (1977). All these models have the advantage that they can be used to value all interest-rate-contingent claims in a consistent way. Their major disadvantages are that they involve several unobservable parameters and do not provide a perfect fit to the initial term structure of interest rates.

According to the one-factor models, all the information about the term structure at any point in time can be summarized by one factor. The short term interest rate could be chosen for this single factor models. In consequence, only the short term interest rate and the time to maturity will affect the price of any interest rate contingent claim.

All models of one-factor start by specifying the stochastic differential equation. This equation can be written:

$$dr(t) = f(r,t)dt + \rho(r,t)dW(t)$$

where W(t) is a wiener process, and f represents the drift coefficient, while ρ is the diffusion term. By specifying those functions, many researchers has proposed their own term structure interest rates. The purpose of this section is to propose a very well-known one-factor models that has been used to model the yield curve. Merton, Vasicek, CIR… are researchers who model the yield curve taking a one variable of risk which is the interest rate and especially the short rate. The criteria classification proposed is to mention those models not by consistent but citing them by a chronological order.

1-1- Merton,1973:

One of the more important developments in modern capital market theory is the Sharpe-Lintner-Mossin mean-variance equilibrium model of exchange, called the capital asset pricing model. It is still subject to theoretical and empirical criticism. Because the model assumes that investors choose their portfolios according to the Markovitz mean-variance criterion.

Merton develops an equilibrium model of the capital market which, (i) has the simplicity and empirical tractability of the capital asset pricing model, (ii) is consistent with expected utility maximization and the limited liability of assets, and (iii) provides a specification of the relationship among yields that is more consistent with empirical evidence.

1-2- Vasicek,1977:

Vasicek gives an explicit characterization of the term structure of interest rates in an efficient market. The model is widely used for pricing the bonds. Additionally, it uses the Ornstein-Uhlenbeck process to compute the spot interest rate. This model is a one-factor model which means that rates depend on the spot interest rate. Thus, the spot rate defines the whole term structure. For more development, see the next section.

1-3- Cox, Ingersoll and Ross,1985:

Cox, Ingersoll and Ross use an intertemporal general equilibrium asset pricing model to study the term structure of interest rates. In their model, anticipations, risk aversion, investment alternative and preferences about the timing of consumption all play a role in determining bond prices.

The researchers propose a description of the term structure interest rates as a stochastic differential equation described as follows: $dr = k(\theta - r)dt + \sigma\sqrt{r}dz$ for $k, \theta > 0$, this corresponds to a continuous time first-order autoregressive process where the randomly moving interest rate is elastically pulled toward a central location or long term value, θ. The parameter k determines the speed of adjustment. The interest rate behavior implied by this structure thus has the following empirically properties: (i) Negative interest rate are precluded. (ii) if the interest rate reaches zero, it can subsequently become positive. (iii) the absolute variance of the interest rate increases when the the interest rate itself increases. (iv)There is a steady state distribution for the interest rate.

1-4- Ho, Lee, 1986:

Ho et. al. proposes an alternative approach to pricing models. The approach is taking the term structure as given, and deriving the feasible subsequent term structure movements. These movements must satisfy certain constraints to ensure that they are consistent with an equilibrium framework. Specifically, the movements cannot permit arbitrage profit opportunities. They called these interest rate movements arbitrage-free rate movements (AR). When the AR movements are determined, the interest rate contingent claims are then priced by the arbitrage methodology, which is used in CIR. Therefore, their model is a relative pricing model in the sense that they price the contingent claims relative to the observed term structure; however, they do not endogenize the term structure as the CIR model do. Thus, Ho and Lee pioneered a new approach by showing how an interest rate model can be designed so that it is automatically consistent with any specified initial term structure.

1-5- Hull, White, 1990:

Hull and White show that the one-state-variable interest rate models of Vasicek (1977) and Cox, Ingersoll, and Ross (1985) can be extended so that they are consistent with both the current term structure of interest rates and either the current volatilities of all spot interest rates or the current volatilities of all forward interest rates. The extended Vasicek model is shown to be very tractable analytically. They compare option prices obtained using the extended Vasicek model with those obtained using a number of other models.

Section 2: the Vasicek model:

The yield to maturity $R(t,T)$ is the internal rate of return at time t on a bond with maturity date t,T=S.

$$P(t,T)e^{(T-t)R(t,T)} = 1$$

This relation can be written:

$$R(t,T) = -\frac{LnP(t,T)}{T-t}$$

The rates $R(t,T)$ considered as a function of T will be referred to as the term structure at time t.

The spot rate as the instantaneous borrowing and lending rate:

$$r(t) = R(t,0) = \lim_{T\to 0} R(t,T)$$

At any time the current value $r(t)$ of the spot rate is the instantaneous rate of increase of the loan value.

It is assumed the $r(t)$ is a stochastic process, subject to two requirements: first, $r(t)$ is a continuous function of time. Second, $r(t)$ follows a Markov process.

Process that are Markov and continuous are called diffusion process.

All in all, they can be described by an Ito stochastic differential equation of the form:

$$dr(t) = f(r,t)dt + \rho(r,t)dz$$

Where $z(t)$ is a wiener process with incremental variance dt. The functions $f(r,t)dt$, $\rho^2(r,t)$ are the instantaneous drift and variance, respectively, of the process (t) .

Most of the diffusion models for interest rates are based on the no-arbitrage principle and they are characterized by similar assumptions on the bond market:

(A.1): the single variable that determines the state of economy at time t is the spot rate $r(t)$;

(A.2): the spot rate follows a diffusion process;

(A.3): the market is efficient: there are no transactions costs, information is available to all investors and every investor acts rationally.

Vasicek illustrates the general model by assuming that:

The market price of risk $q(t,r) = q$ is a constant, independent of the calendar time and the level of the spot rate. In addition, the spot rate $r(t)$ follows the Ornstein-uhlenbeck process:

$$dr(t) = \alpha\big(\gamma - r(t)\big)dt + \sigma dz$$

The Ornstein-uhlenbeck process with $\alpha > 0$ is sometimes called the elastic random walk. It is a Markov process with normally distributed increments. The instantaneous drift $\alpha\big(\gamma - r(t)\big)$ represents a force that keep pulling the process towards it's long term mean γ with magnitude proportional to the deviation of the process from the mean. The stochastic element, which has a constant instantaneous variance σ^2, causes the process to fluctuate around the level γ in an erratic, but continuous, fashion.

According to given assumptions in this model, the value for the value P of the zero-coupon bond at time t with maturity at time T could be expressed as follows:

$$P\big(t,T,r(t)\big) = exp\left[\frac{1}{\alpha}\big(1 - e^{-\alpha(T-t)}\big)(R_\infty - r(t)) - (T-t)R_\infty - \frac{\sigma^2}{4\alpha^3}(1 - e^{-\alpha(T-t)})^2\right]$$

Where: $R_\infty = \gamma + \frac{\sigma q}{\alpha} - \frac{1}{2}\frac{\sigma^2}{\alpha^2}$

And the term structure of interest rates takes the form:

$$R(t,T) = R_\infty + (r(t) - R_\infty)\frac{1}{\alpha T}(1 - e^{-\alpha T}) + \frac{\sigma^2}{4\alpha^3 T}(1 - e^{-\alpha T})^2$$

Since $r(t)$ is normally distributed by virtue of the properties of the Ornstein-uhlenbeck process, and $R(t,T)$ is a linear function of $r(t)$, it follows that $R(t,T)$ is also normally distributed.

It will only be noted that the differential equation imply that the discrete rate series, follows a first order linear normal autoregressive process of the form:

$$r_t = c + a(r_{t-1} - c) + \varepsilon_t$$

With independent residuals ε_t. The process is the discrete elastic random walk, fluctuation around lts mean c. The parameters $c, a \; and \; s^2 = E(\varepsilon_t^2)$ could be expressed in terms of $\gamma, \alpha, \sigma, q$. The constant a, which characterizes the degree to which the next term in the series $[R_t]$ is tied to the current value, is given by $a = e^{-\alpha T}$.

Section 3: Vasicek model's parameters estimation:

3-1- Data:

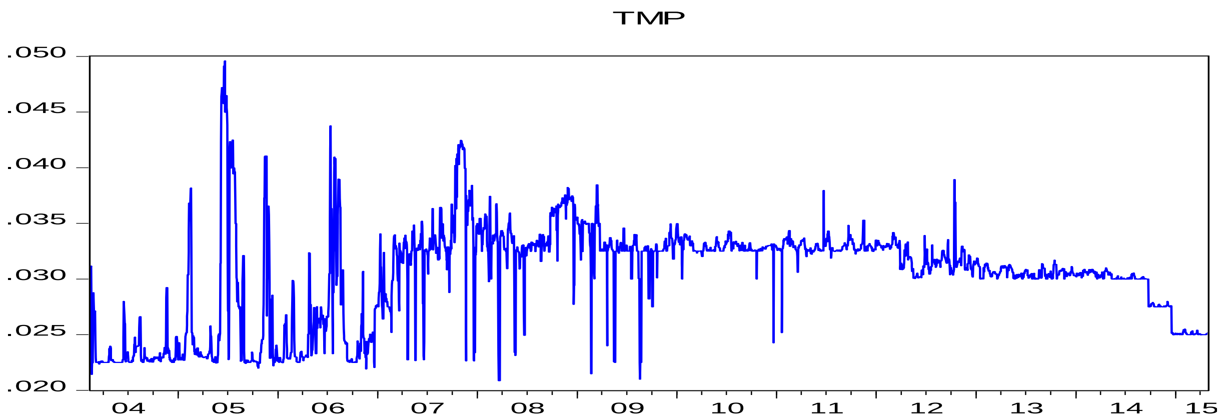
To generate the Moroccan yield curve, the data of short interest rates from money market and the data of prices of coupon-zero bond are used. Using linear least square method in order to estimate parameters of the Vasicek model from 2004 to 2015 and inserting them into the term structure formula. It is collected from Bank-al-Maghreb (Moroccan central bank) that has been used as a typical short-term interest rate.

Basic statistics of daily weighted average rates from 2004 through 2015 are summarize below:

N	Mean	Std.dev	Minimum	Maximum	Skewness	Kurtosis
4095	0.030266	0.004559	0.02087	0.04959	-0.07621	3.265581

Source: Eviews

Figure: daily evolution of interbank of Moroccan money market since 2004 to 2015



The correlogram of data of interest rates shown that we could presume that the data follow a first-order linear autoregressive process. The statistic of Jarque-Bera (15.99) confirm that the process follows a Gaussian process. This result confirms the assumption of Vasicek model (1977). We conclude that our data follows a first-order linear autoregressive process.

3-2- Unit root test (Dickey-Fuller):

The augmented Dickey-Fuller (ADF) test applied to the sample of data reject the null hypothesis of a unit root at the 5% risk level. The result of this test is summarized below.

Result of unit root test:

Model	ADF test statistic	Test critical value (5%)	Decision
With trend and intercept	-9,398229	-3,410903	Stationary
With intercept	-9,044416	-2,862050	Stationary
None	-1,393970	-1,940901	Non stationary

Source: Eviews

3-3- Parameters estimation:

In this sub-section, we have to estimate the parameters of the Vasicek model using the ordinary least square (OLS). Longstaff and al (1992) estimated a variety of continuous-time models of the short-term riskless rate using the Generalized Method of Moments (GMM). In a paper published in 2013, Fatma Chakron and Fathi Abid, The researchers develop a methodology to estimate the interest rates yield curve and its dynamics in the Tunisian bond market using OLS and Maximum likelihood estimation.

We estimate the parameters of the continuous-time model using a discrete-time econometric specification, according to Brennan-schwartz (1982), dietrich-campbell and others did.

The exact discrete model corresponding to the stochastic differential equation is a first-order autoregressive AR (1) model:

$$r_t - r_{t-1} = c + dr_{t-1} + \varepsilon_t$$

$E(\varepsilon_t) = 0 \quad ; \quad E(\varepsilon_t^2) = \sigma^2 r_t^{2\gamma}$

This discrete-time model has the advantage of allowing the variance of interest rate changes to depend directly on the level of the interest rate in a way consistent with the continuous-time model. It's important to acknowledge that the discretized process above is only an approximation of the continuous-time specification.

Where:

$$\alpha = -log\hat{d}$$

$$\gamma = \frac{\hat{c}}{1 - e^{-a\Delta t}} = \frac{\hat{c}}{1 - \hat{d}}$$

$$\sigma^2 = \frac{s^2 * 2a^3 * \theta^2}{(1-e^{-aT})^2(1-\hat{d}^2)} = s^2\sqrt{\frac{-log\hat{d}}{1-\hat{d}^2}}$$

The estimation result:

Rate	$\hat{c}$	Stat t	$\hat{d}$	Stat t	α	γ	σ^2
TMPjj	-0.001073 (0.000126)	-8,5195	0.035390 (0.004113)	8.604079	3.34	-0.001	0.0107

Source: Eviews

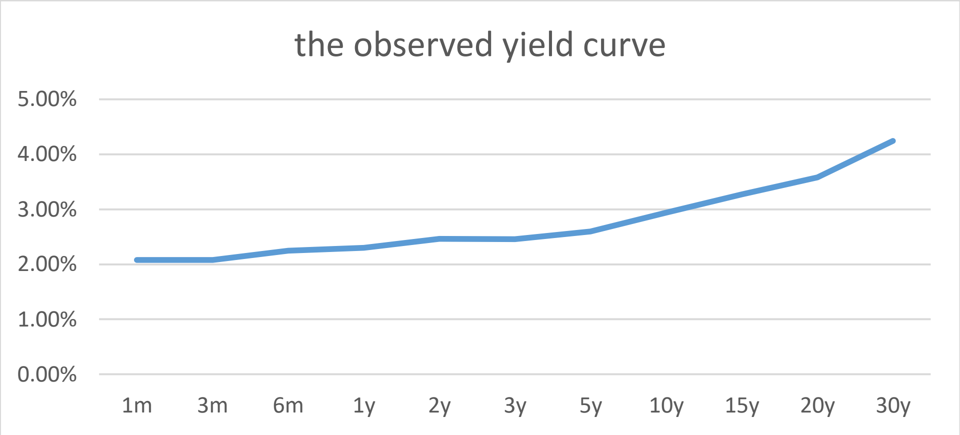
The standard errors of the estimated coefficient are in parentheses.

All parameters of the model are significant at the 5% risk level. We observe that the conditional volatility of the process is very weak which is mean that is not highly sensitive to the level of the short-term yield. In fact, the most relevant problem in the present context is the estimation bias in the practical use of econometric estimators for the continuous-time discretized models. In general, discretization introduces an estimation bias since the internal dynamics between the sampling points are ignored.

4- Vasicek model of the term structure of interest rates:

4-1- The observed yield curve:

We take as reference the yield curve published by the Moroccan central bank in 28/07/2016. The shape of the term structure of the interest rates is an upward sloping yield curve because short-term interest rates are below long term interest rates. In other words, longer-term interest rates are usually higher than shorter term interest rates that why we are saying normal yield curve. Th graph below summarize the shape of the term structure of interest rates for treasury bills in 28/07/2016.



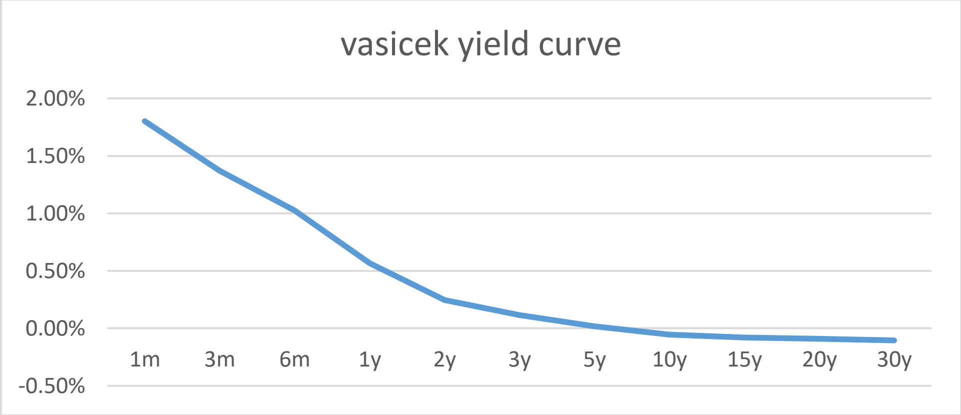
4-2- Vasicek yield curve:

By choosing 28/07/2016 as a reference where the short-term of interest rate is 2.263%, we construct the Vasicek yield curve using:

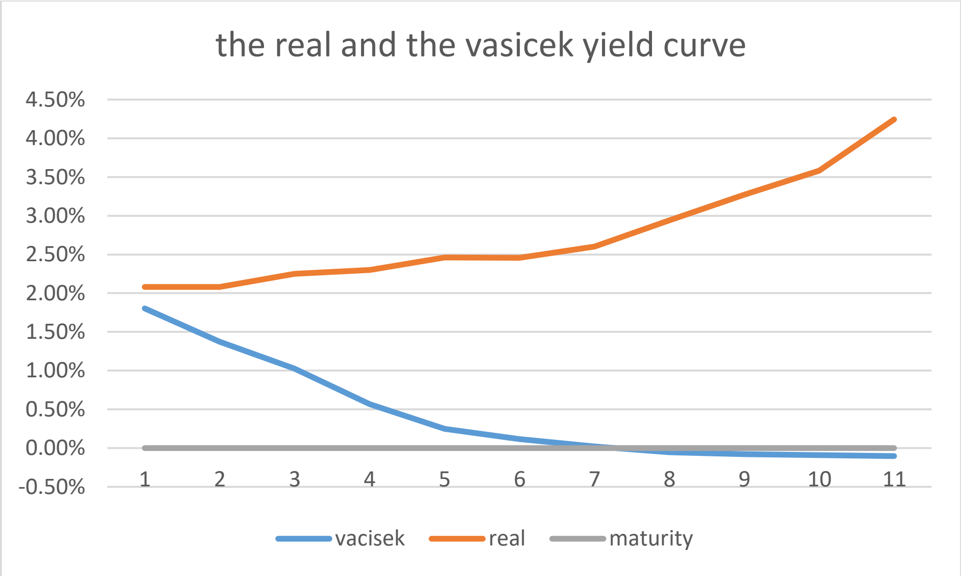
$$R(t,T) = R_\infty + (r(t) - R_\infty)\frac{1}{\alpha T}(1 - e^{-\alpha T}) + \frac{\sigma^2}{4\alpha^3 T}(1 - e^{-\alpha T})^2$$

where : $$R_\infty = \gamma + \frac{\sigma q}{\alpha} - \frac{1}{2}\frac{\sigma^2}{\alpha^2}$$

In a neutral-risk we assume that: q=0, well, $R_\infty = \gamma - \frac{1}{2}\frac{\sigma^2}{\alpha^2}$



The shape of the term of structure of interest rates using Vasicek model is a downward sloping yield curve as shown in the graph above, which is not connected with the real yield curve.



The shape of Vasicek model and the shape of the yield curve of Moroccan market are not similar. Many reasons could explain that difference. Firstly, our work does not take in consideration the market risk, because we are working in a neutral risk space. And also it is hard to observe it. Secondly, the bias estimation as a result to the passage from a continuous-time to a discretized-time.

5- Conclusion:

This paper is an attempt to model the term structure of interest rates, which measures the relationship among the yields on default-free securities that differ only in their term to maturity. The determinants of this relationship have long been a topic of concern of economists and practionners. We opt for the Vasicek model, which is a single-factor model the term structure of interest rates in a continuous-time. To estimate the model's parameters, we use data from the Moroccan central bank and the ordinary least square that is an econometric technique is used. The choice of this technique is due to discretization of the diffusion model, which is a linear, and a first-order autoregressive model. Then we infer the result to estimate the model's parameters. After that, we construct the Vasicek model and compare it with the real yield curve extracted from the Moroccan central bank. The result shows that Vasicek model does not fit the actual yield curve.

This result may let us think about an alternative for using another model like CIR models or other models cited above in second section. And also maybe if we use a jackknife estimation to include the bias estimation.

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